Fostering the Promise of High Achieving Mathematics Students through Curriculum Differentiation

Simone Zmood
Monash University
<szmo1@student.monash.edu>

Recent research suggests some teachers may not have a wide range of teaching and learning strategies for their most proficient mathematics students, which could impact on these students' learning and ongoing improvement in performance. This paper outlines the different drivers of high achievement and explores the main curriculum differentiation strategies schools and teachers can use for such students. With a toolkit of appropriate strategies, teachers can ensure that class time is productive for their high achieving students and that these students have the opportunity to fully develop their mathematical abilities over the course of the year.

Introduction

From television and sport, it seems that the community at large is interested in elite performance and the idiosyncratic. Even at school, precociousness is celebrated in art, poetry and music, yet this often does not extend into fostering the promise of high achieving mathematics students. The objective of a recent longitudinal study by Griffen (2012) of school students' reading comprehension and mathematics performance at Victorian Catholic and government schools was "to enable teachers to use data within a developmental framework to improve performance of *all* students" (p. 77). It was centred on the hypothesis "that if the teachers targeted instruction where students were most ready to learn, improvements in performance would be pronounced" (p. 77).

Griffen (2012) found that whilst "students at the bottom levels of the proficiency are improving rapidly … [s]tudents at the top end of the scale are hardly improving at all" (p. 84). These results run counter to the logic that the more proficient mathematics students would be higher ability and thus "should be able to improve at a faster rate than those at lower levels" (p. 84). Another key result was the scarcity of strategies identified by teachers for highly proficient students which might be directly influencing this performance and Griffin presents a number of "possible explanations for a lack of strategies at higher order skill levels" (p. 83).

Of course, it is possible that this scarcity of strategies is a limitation of the assessment system. Nevertheless, the issue is worth exploring because without appropriate learning experiences advanced learners may become mentally lazy and extrinsically motivated; they may become perfectionists who fear failure, play it safe and hence do not develop a sense of self-efficacy. They may also not develop skills for studying and coping with life's adversities (Tomlinson, 2001). Thus advanced learners need "teachers that coach for growth and curriculums that are appropriately challenging" (Tomlinson, 2001, p. 11) in order to reach their potential. A fundamental assumption of the following argument is that there would be benefits if teachers had strategies and resources at their disposal to enable them to maximise the potential of all students in their class, including their best students.

2014. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). Curriculum in focus: Research guided practice *(Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia)* pp. 677–684. Sydney: MERGA.

In this paper, 'high achieving' refers to those students who demonstrate a high level of mathematics proficiency in class as well as on state-wide or national testing. The proportion of students achieving at a 'high' level within a class or year level may differ from the proportion performing at a high level relative to the broader student population depending on the particular cohort of students in a year level or school. Numerous researchers have made the case for considering and catering to the learning needs of high achieving and gifted mathematics students (Assouline & Lupkowski-Shoplik, 2011; Diezmann & Watters, 2002; Stanley & George, 1980).

The second issue is that once high achieving students have been identified decisions need to be made on how to respond. Differentiation or differentiated instruction acknowledges that some students in a class may benefit most from a modified mathematics curriculum with changes to either, or both, curriculum **content** which is "*what* is to be taught and learnt" (ACARA, 2013, n.p.) and **delivery** or "*how* content is explored or developed, that is, the thinking and doing of mathematics" (ACARA, 2013, n.p.). Put succinctly, "we should strive for optimal matches between the available resources and each learner's profiles of strengths and weaknesses, passions and preferences, within and across domains of knowledge" (Kanevsky, 1995, p. 162).

The first section of this paper presents a model of learning potential to help develop an understanding of the drivers of achievement in mathematics. The next section looks at the different sources of mathematical potential and sheds further light on the characteristics of high achieving mathematics students. The third section offers a range of strategies which teachers and schools can consider and links these back to student characteristics to help determine which strategies best address the needs of different clusters of high achieving students.

Drivers of Achievement in Mathematics

A key issue is what factors are driving students' high achievement. Kanevsky's (1995) model of learning potential, illustrated in Figure 1, provides a useful framework for discerning the particular combination of factors which contribute to high achievement in mathematics for an individual student.

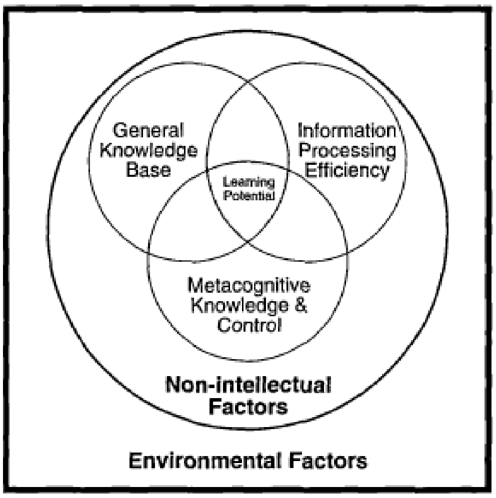

Figure 1. Factors contributing to individual differences in learning potential (Kanevsky, 1995, p. 158).

Learning potential is considered to be the dynamic interaction of two internal factors and one external factor: (1) intellectual, (2) non-intellectual, and (3) environmental. The intersection of the three intellectual factors—general knowledge base, information processing efficiency, and metacognition—represents a student's intellectual potential. As Kanevsky (1995) notes,

> learners with an advantage in one, two or all of these areas have the potential to learn more effectively than their peers'… [but ultimately] a learner's cognitive competence will be limited by the quality of the weakest element in this dynamic system, regardless of how well-developed the other elements may be. (p. 160)

According to this model, learning potential is amplified or inhibited by the interaction of the internal and external factors. As well as the three intellectual factors, a range of internal non-intellectual factors will affect learning including feelings, interests, attitudes, global self-concept and domain-specific self-perceptions. Environmental factors encompass the influence of people such as parents, teachers and peers; the learning environment such as time of day and learning materials; and classroom management such as instruction mode, grouping strategies and seating arrangements.

A high achieving student is *visibly good* at mathematics, but this raises a whole host of questions. Is the student high achieving because they find school mathematics easy (i.e., existing knowledge base or efficient information processing capability)? Or perhaps the student enjoys the challenge of mathematics and is prepared to work hard (i.e., non-intellectual)? Is the student high achieving because they are being coached, or "hot housed" by parents (i.e., knowledge base and environmental factors)? Or do they truly have gifted reasoning capabilities (i.e., information processing efficiency and well-developed metacognition)? Is the student intrinsically motivated to learn mathematics and do well (i.e., non-intellectual factors), or extrinsically motivated to please others such as parents and teachers (i.e., non-intellectual and environmental factors)? Is the student growing up in a home or school environment that values mathematics (i.e., non-intellectual and environmental factors)?

Clearly cognitive, affective and environmental factors can each play a part in students' school performance. Research has shown that intellectual potential correlates highly with academic achievement (Lubinski & Benbow, 2006). It thus becomes important to differentiate between high achieving students with high intellectual potential in mathematics and those who have more typical intellectual potential but may be high achieving because of other factors such as motivation. Whilst both types of high achievers would benefit from differentiated programs, the type of program needed by each group will vary.

Sources of Potential in Mathematics

Mathematical potential is also often referred to as mathematical ability, talent or promise. Firstly, it should be recognised that "mathematical talent is not a single construct, and different students display different types of mathematical talent at different times" (Gavin, 2005, p. 25). Krutetskii (1976) studied the mathematical abilities of schoolchildren and identified nine abilities of mathematically gifted children which were categorised by Usiskin (1999) into four major factors: formalisation, logical thought, curtailment and flexibility. Formalisation involves seeing the overall structure of a mathematical problem and being able to generalise. Logical thought processes filter incoming data to look at the problem as a whole and gifted students can skip steps (curtailment) to reach the solution. Gifted mathematical students are flexible and can change strategies easily and frequently to solve a problem (Krutetskii, 1976; Usiskin, 1999). It should be noted that Krutetskii (1976) specifically mentioned several characteristics which are useful but not necessary characteristics of mathematical giftedness, including swiftness, computational ability, and memory for facts and formulas. Ironically, these are the characteristics most visible to teachers and are likely to lead to a student being identified as *good* at mathematics.

Usiskin's (2000) hierarchy of mathematical talent emphasises the increasing importance of creativity needed to reach the top of the field of mathematics. Geake (2009) concurs, writing that "creative mathematical thinking requires extensive and thorough mathematical knowledge … [and] gifted mathematical thinking involves a high degree of creativity" (p. 161).

It should be noted that there is no agreed definition of giftedness despite numerous theories and models. The Pentagonal Theory of identifying giftedness is based on five criteria: excellence, rarity, productivity, demonstrability and value (Sternberg, Jarvin, & Grigorenko, 2011). Sternberg, Jarvin and Grigorenko (2011) say that for a person to be considered gifted, he or she must demonstrate excellence through "valid assessments" to a rare level of actual or potential productivity in a "dimension or set of dimensions" that is "valued by his or her society" (p. 27).

A central purpose for identifying high ability mathematics students is to design appropriate learning experiences for them (Gavin, 2005). Resources available to deliver these learning experiences are an important consideration in the definition of giftedness. The rarity criterion in the above-mentioned Pentagonal Theory acknowledges that "we seek out rarity in part because of our inability to serve all students who may truly have very impressive potentials" (Sternberg et al., 2011, p. 9). Thus there is no absolute cut-off for giftedness and we choose an arbitrary percentage of children for a special mathematics program based on our values and resources. Typically, between two and ten percent of students are labelled as mathematically gifted based on their results on standardised psychometric testing (Gagné, 2004; Sternberg et al., 2011). However, it is the contention of this paper that understanding the drivers of an individual student's achievement and the unique mix of abilities which influence his or her mathematical potential is a more effective way of determining the appropriate mathematical program rather than a simple number.

Strategies for Differentiating Curriculum

Practices in gifted education are often acknowledged as being good teaching practice in general and are thus appropriate for consideration for all high achieving mathematics students. Maker and Schiever (2010) suggest developing a qualitatively different curriculum by adapting the learning environment, content, process and product. Optimal learning for students occurs when they are learning in Vygotsky's (1978) zone of proximal development which is the difference between independent learning and guided learning. Gavin (n.d.) emphasises that "if students are given "enrichment sheets" to work on independently and can do this successfully, it usually means the material is NOT challenging for them" (para. 9). According to Sheffield (1999), this challenge can be introduced into the curriculum along three dimensions: pace, breadth and depth.

Introducing Challenge through Curriculum Pace

One possible approach to increasing the challenge of a mathematics program is by increasing the pace, usually referred to as acceleration. Acceleration content is typically not differentiated and is the same skills-based material found in the standard mathematics curriculum. Students move more quickly through the curriculum and receive formal recognition, or credit, for attained mastery. Understanding of each student's underlying mathematical abilities is important to ensure that the student will not suffer undue stress from attempting to maintain an unrealistic pace (Assouline & Lupkowski-Shoplik, 2011), which could ultimately detract from their enjoyment and progress in mathematics.

The degree of acceleration can be set by a teacher or individualised with self-paced learning. A teacher may choose to compact one year's curriculum allowing time for options such as a specially designed enrichment and extension curriculum or independent projects in the remainder of the academic year. In this situation students' learning will align with the core mathematics curriculum year-on-year, but with time for other things as well. Alternatively, the curriculum may be accelerated continuously so that these students move ahead of their age peers in the standard curriculum. Telescoping, compressing the first four years of high school into three, is used by Victoria's Selective Entry Accelerated Learning (SEAL) schools years (Kaman & Kronborg, 2012) and also by schools that offer an accelerated mathematics stream.

The Study of Mathematically Precocious Youth (SMPY) started in the 1970s and is the largest ongoing longitudinal study of high ability students in mathematics. Selected students were offered special summer programs based on the Diagnostic Testing → Prescriptive Instruction (DT→PI) model of pre-testing the student's current knowledge to identify gaps and then using mentoring and small group lectures to achieve mastery (Stanley & George, 1980). This allows each student to progress at his or her own pace and creates a rapid iterative cycle of progress through testing and then instruction. Research has found immediate and long term benefits of SMPY acceleration on academic, social-emotional and career progress (Lubinski & Benbow, 2006).

Assouline and Lupkowski-Shoplik (2011) provide detailed guidance on how the DT→PI model can be adapted to provide an individualised, classroom-based, self-paced accelerated mathematics program. This approach could be used in any classroom provided the class teacher is willing to support such an arrangement. The key issue is to ensure that a multi-year curriculum plan is put in place with appropriate support and resources for the teachers who will implement it and careful thought given to handling the final years of schooling when the student has already completed the standard mathematics curriculum.

Introducing Challenge through Curriculum Breadth

Another approach to adding challenge and interest to the mathematics curriculum is by adding breadth, which is often referred to as enrichment. When considering the content of the intended curriculum, it should be acknowledged that the field of mathematics is complex with many interrelated areas and that the school mathematics curriculum is intentionally designed in a sequential manner, where each new concept builds on previous concepts. Since the standard school curriculum only covers a small proportion of the extensive field of mathematics, additional breadth may be achieved by studying extra areas of mathematics directly related to core content or by interdisciplinary studies. It is important that "mathematically talented students should study a core curriculum presented in a systematic manner, not a random assortment of enrichment topics" (Assouline & Lupkowski-Shoplik, 2011, p. xxiv).

One example is Georgia's Project for Gifted Education in Math and Science (Project Ga-GEMS), a Year 9-10 program which offered an integrated hands-on mathematics and science curriculum focusing on higher-level thinking skills and real-life laboratory experiences (Tyler-Wood, Mortenson, Putney, & Cass, 2000). Research showed significant academic differences at the end of the two-year program and these benefits were maintained until the end of high school (Tyler-Wood et al., 2000) offering "empirical support for the use of authentic integrated mathematics curricula that make rich connections with the sciences and relevant real-world applicable learning opportunities" (Sriraman & Steinthorsdottir, 2008, p. 403).

Introducing Challenge through Curriculum Depth

A third approach to adding challenge is by increasing curriculum depth or complexity, often referred to as extension. Sheffield (1999) warns that "in many cases, both enrichment and acceleration programs have missed the mark; if the curriculum is "a mile wide and an inch deep", it does not help to widen the curriculum even further or ask the students to run the mile faster" (p. 45). Instead of a shallow look at mathematical topics, Sheffield (2003) recommends: "Think deeply about simple things! … [asserting that] the most complex and enjoyable mathematical explorations begin with simple concepts that [can be mined] for their richness and elegance" (p. 8). This is the underlying premise behind Singapore's pentagonal mathematics framework centred on problem solving (Leong et al, 2011) and Australian research on teaching mathematics through representational, contextual and open-ended tasks (Sullivan, Clarke, & Clarke, 2013). Three to eight levels of differentiated tasks form the core of the student-centred content lessons of the Sunshine College Years 7-9 mathematics program (Parsons & Reilly, 2011), which has enabled students to achieve significantly higher relative growth than the state average on NAPLAN testing (Preiss, 2013).

Project M^3: Mentoring Mathematical Minds is a research-validated differentiated curriculum for Years 3-5 which emphasises the development of thinking and acting like professional mathematicians utilising critical and creative thinking for problem solving expressed through both verbal and written communication (Gavin, Casa, Adelson, Carroll, & Sheffield, 2009). The U.S. study was based on ability-grouped classes for three consecutive years and demonstrated significant beneficial differences in conceptual reasoning on standardised tests relative to control groups (Gavin et al., 2009); however, the resources developed could be used as enrichment for the standard classroom curriculum or replacement of current units in any class.

Selecting Appropriate Differentiation Strategies Based on Student Characteristics

Whilst challenge can be introduced to the mathematics curriculum along three dimensions—pace, breadth and depth—these are not necessarily mutually exclusive. Assouline and Lupkowski-Shoplik (2011) make the point that "mathematics builds upon itself so that, in reality, it is extremely difficult to "enrich" a student without actually accelerating his or her study of mathematics" (p. 6). Sheffield (2003) believes that an optimal mix and level of each dimension should be found "but it is more critical to add depth and complexity to the study of mathematics" (p. 8). Assouline and Lupkowski-

Shoplik (2011) suggest high achieving students should be tested on an at-level mathematical aptitude test and those that score above the 95^{th} percentile should be tested again at least two years above-level. Those students, who are performing like or better than the average student two years above their school level (above 50^{th} percentile), are suitable candidates for acceleration, whilst the remaining students should receive enriched mathematics instruction.

Conclusion

Clearly, there are many different ways of supporting high achieving students and fostering their mathematical promise. If we are to design and implement appropriate differentiation strategies for high achieving students it is important to understand the unique mix of cognitive, affective and environmental factors which impact each student.

Encouraging depth and complexity of understanding through mathematical tasks and problem solving will help high achieving students to move along the continuum of mathematical capability beyond being problem solvers to become problem posers and creators of original solutions (Sheffield, 1999, 2003). Gifted mathematics students may be able to pursue these deeper mathematical explorations at a faster pace and generalise further, and acceleration is an option to consider. Enrichment or 'breadth' offers all high achieving students the opportunity to link their mathematics learning to other areas of mathematics or across disciplines. High achieving students who are not gifted should be offered enrichment in areas of interest to encourage ongoing enjoyment of mathematics.

Differentiation is a multi-step and multi-faceted process which can invoke trepidation and resistance amongst teachers, but there are fully-developed differentiated curricula, lesson plans, mathematics enrichment books and online resources which teachers can utilise. Teachers do not need to re-invent the wheel or embark on differentiation alone. Whilst teachers ideally develop skills in differentiation during their pre-service courses and ongoing professional development, it is also important that schools provide resources and support, such as mentoring, so that teachers can access existing differentiated mathematics materials, develop their own resources and build confidence in delivering differentiated classroom teaching to ensure meaningful learning experiences for all the students in their class. Using differentiation strategies can assist teachers in guiding high achieving students to improve their performance at an appropriate rate and thus increase the likelihood of reaching their potential.

Finally, it should be acknowledged that "the development of mathematical potential, like any other valued ability, is something that takes dedication and hard work on the part of teachers, parents and the students themselves" (Sheffield, 2003, p. 1). Fostering mathematical promise is a shared responsibility.

References

Assouline, S. G., & Lupkowski-Shoplik, A. (2011). *Developing math talent: A comprehensive guide to math education for gifted students in elementary and middle school* (2nd ed.). Waco, TX: Prufrock Press.

Australian Curriculum, Assessment and Reporting Authority (ACARA). (2013). *The Australian curriculum: Mathematics*. Retrieved from http://www.australiancurriculum.edu.au/Mathematics/Content-structure

Diezmann, C. M., & Watters, J. J. (2002). Summing up the education of mathematically gifted students. In B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. Thomas (Eds.), *Mathematics education in the South Pacific* (Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia, pp. 219-226). Auckland: MERGA.

Gagné, F. (2004). Transforming gifts into talents: The DMGT as a developmental theory. *High Ability Studies*, *15*(2), 119-147.

Gavin, M. K. (n.d.). *Meeting the needs of talented elementary math students*. Neag Center for Gifted Education and Talent Development. Retrieved from http://www.gifted.uconn.edu/projectm3/meeting_the_needs .html

Gavin, M. K. (2005). Are we missing anyone? Identifying mathematically promising students. *Gifted Education Communicator*, *36*(3–4), 24–29.

Gavin, M. K., Casa, T. M., Adelson, J. L., Carroll, S. R., & Sheffield, L. J. (2009). The impact of advanced curriculum on the achievement of mathematically promising elementary students. *Gifted Child Quarterly*, *53*(3), 188-202.

Geake, J. G. (2009). *The brain at school: Educational neuroscience in the classroom*. Berkshire, England: Open University Press.

Griffin, P. (2012, August). The influence of teaching strategies on student achievement in higher order skills. In *ACER 2012 Research Conference: School improvement: What does research tell us about effective strategies*?, (pp. 77-82). Sydney, Australia.

Kaman, Y. & Kronborg, L. (2012). Perceptions of learning at a select entry accelerated high school for high ability students. *Australasian Journal of Gifted Education*, *21*(2), 47-61.

Kanevsky, L. (1995). Learning potentials of gifted students, *Roeper Review*, *17*(3), 157-163.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in schoolchildren*. Chicago, MI: University of Chicago.

Leong, Y. H., Dindyal, J., Toh, T. L., Quek, K. S., Tay, E. G., & Lou, S. T. (2011). Teacher preparation for a problem-solving curriculum in Singapore. *ZDM Mathematics Education*, *43*(6-7), 819-831.

Lubinski, D., & Benbow, C. P. (2006). Study of mathematically precocious youth after 35 years: Uncovering antecedents for the development of math-science expertise. *Perspectives on Psychological Science*, *1*(4), 316-345.

Maker, C. J., & Schiever, S. W. (2010). *Curriculum development and teaching strategies for gifted learners* (3rd ed.). Austin, TX: Pro-Ed.

Parsons, J., & Reilly, Y. (2011). *Sunshine College 2011 numeracy program*. Retrieved from www.nzamt.org.nz/2011_con_resources/handout.doc

Preiss, B. (2013, September 4). Results add up for maths with a difference. *The Age*. Melbourne, Australia: Fairfax Media. Retrieved from http://www.theage.com.au/victoria/results-add-up-for-maths-with-a-difference-20130903-2t393.html#ixzz2hYCZzQGV

Sheffield, L. J. (1999). Serving the needs of the mathematically promising. In L.J. Sheffield (Ed.), *Developing mathematically promising students* (pp. 43-55). Reston, VA: National Council of Teachers of Mathematics.

Sheffield, L. J. (2003). *Extending the challenge in mathematics: Developing mathematical promise in K-8 students*. Thousand Oaks, CA: Corwin Press.

Sriraman, B. & Steinthorsdottir, O.B. (2008). Mathematics, Secondary. In J.A. Plucker & C.M. Callahan (Eds.), *Critical issues and practices in gifted education* (pp. 395-407). Waco, TX: Prufrock Press.

Stanley, J. C. & George, W. C. (1980). SMPY's ever-increasing D4. *Gifted Child Quarterly*, *24*(1), 41-48.

Sternberg, R. J., Jarvin, L., & Grigorenko, E. L. (2011). *Explorations in giftedness*. New York, NY: Cambridge University Press.

Sullivan, P., Clarke, D., & Clarke, B. (2013). *Teaching with tasks for effective mathematics learning*. New York, NY: Springer.

Tomlinson, C.A. (2001). *How to differentiate instruction in mixed-ability classrooms* (2nd ed.). Alexandria, VA: Association for Supervision and Curriculum Development.

Tyler-Wood, T. L., Mortenson, M., Putney, D., & Cass, M. A. (2000). An effective mathematics and science curriculum option for secondary gifted education. *Roeper Review*, *22*(4), 266-269.

Usiskin, Z. (1999). The mathematically promising and the mathematically gifted. In L.J. Sheffield (Ed.), *Developing mathematically promising students* (pp. 57-69). Reston, VA: National Council of Teachers of Mathematics.

Usiskin, Z. (2000). The development into the mathematically talented. *Journal of Secondary Gifted Education*, *11*(3), 152-162.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Cambridge University Press.